MECH105
Homework 2
MathCAD

Basic calculations - solution to quadratic equation: $a \cdot x^2 + b \cdot x + c = 0$

$a := 1 \qquad b := 2 \qquad c := 3$

$$x_{1st} := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \qquad x_{2nd} := \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

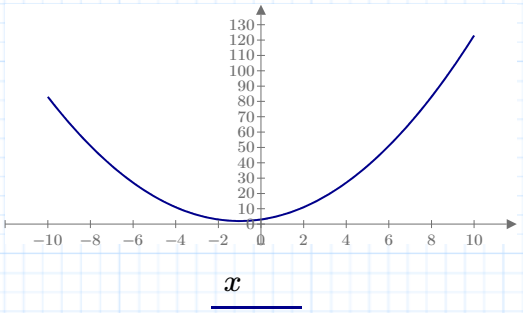
$x_{1st} = -1 + 1.414\mathrm{i} \qquad x_{2nd} = -1 - 1.414\mathrm{i}$

checking results:

$f(x) := a \cdot x^2 + b \cdot x + c$

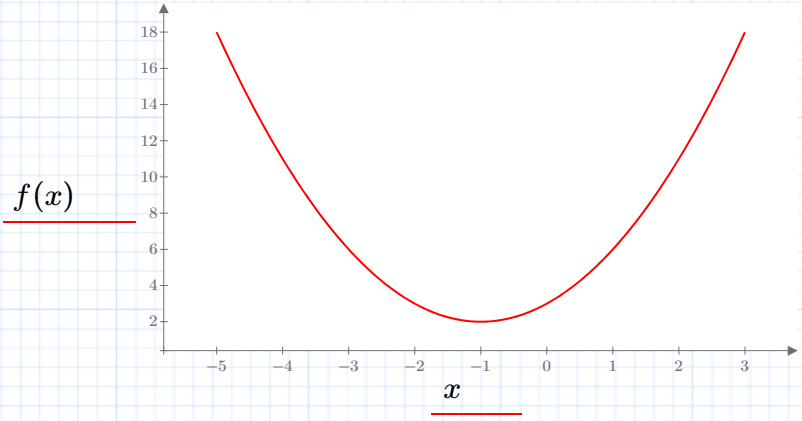
$f(x_{1st}) = 0 \qquad f(x_{2nd}) = 0$

Plotting a function with automated ranges and number of points (which can be changed)



Plotting a function using a range variable, and changing plot display options:

$x := -5, -4.95 .. 3$



$$x = \begin{bmatrix} -5 \\ -4.95 \\ -4.9 \\ -4.85 \\ -4.8 \\ -4.75 \\ -4.7 \\ -4.65 \\ -4.6 \\ -4.55 \\ -4.5 \\ -4.45 \\ \vdots \end{bmatrix} \qquad x = \begin{bmatrix} \vdots \\ 2.45 \\ 2.5 \\ 2.55 \\ 2.6 \\ 2.65 \\ 2.7 \\ 2.75 \\ 2.8 \\ 2.85 \\ 2.9 \\ 2.95 \\ 3 \end{bmatrix}$$

Using units and significant digit display

$m := 100 \cdot lb$ $v := 60 \cdot mph$ $a := 20 \cdot \frac{ft}{s^2}$

$p := m \cdot v$ $p = (1.217 \cdot 10^3)\ \frac{kg \cdot m}{s}$ $F := m \cdot a$ $F = 62.2\ lbf$

Symbolic algebra

$\mathbf{clear}(x)$ clears previous definition of range variable "x" $x = ?$

$$x(y) := \frac{x}{2 \cdot x - 3 \cdot x \cdot y} = \frac{(x-2)^2}{y+2} \xrightarrow{solve, x} \begin{bmatrix} \frac{6 \cdot y + \sqrt{-((y+2) \cdot (3 \cdot y - 2))} - 4}{3 \cdot y - 2} \\ -\frac{\sqrt{-((y+2) \cdot (3 \cdot y - 2))} - 6 \cdot y + 4}{3 \cdot y - 2} \end{bmatrix}$$

evaluate solutions:

$x(5)_0 = 2 + 0.734\mathrm{i}$ $x(5)_1 = 2 - 0.734\mathrm{i}$

Symbolic calculus

$a = 6.096\ \frac{m}{s^2}$ $a := \frac{a}{\frac{m}{s^2}}$ strip units off numerical value $a = 6.096$

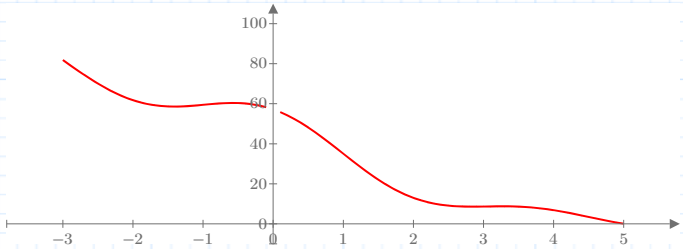
$\mathbf{clear}_{sym}(a)$ allows variable "a" serve as a new symbolic variable, while still keeping its numeric value $a = 6.096$

$$f(x) := (x-a)^2 + \frac{10 \cdot \sin(2 \cdot x)}{x}$$

$$d(x) := \frac{\mathrm{d}}{\mathrm{d}x} f(x) \rightarrow 2 \cdot x - 2 \cdot a + \frac{20 \cdot \cos(2 \cdot x)}{x} - \frac{10 \cdot \sin(2 \cdot x)}{x^2}$$

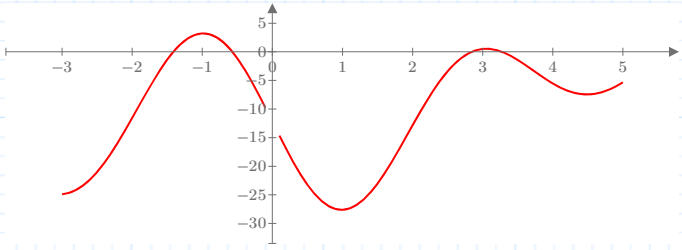
$x := -3, -2.9 .. 5$

$f(x)$



x

$d(x)$



x

Vector and matrix calculations

$v_x := -1 \qquad v_y := -2$

$v := \begin{bmatrix} v_x \\ v_y \end{bmatrix} \qquad v = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$mag := |v| = 2.236 \qquad v \cdot v = 5$ magnitude and dot product

$v := v_x + 1\mathrm{i} \cdot v_y = -1 - 2\mathrm{i}$ complex plane representation

$angle := \arg(v) = -116.565\ °$ angle of a vector (argument)

$v := mag \angle angle = 2.236 \angle -116.565°$ entering and displaying a vector in polar form

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \\ 0 & -2 & 3 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -1.182 & 1.091 & -0.636 \\ 0.545 & -0.273 & -0.091 \\ 0.364 & -0.182 & 0.273 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Programming

$$f(x) := \begin{cases} \text{if } x < 1 \\ \quad x \\ \text{else if } 1 \le x \le 3 \\ \quad -(x-1)^2 + 1 \\ \text{else if } x > 3 \\ \quad -3 \end{cases}$$

piecewise function

$x := -2, -1.9 .. 5$

General programming problem example: find the sum of first N numbers divisible by 3

$N := 10000$

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results := | i ← 0
           | n ← 0
           | total ← 0
           | while n < N
           |     i ← i + 1
           |     remainder ← mod(i, 3)
           |     if remainder = 0
           |         total ← total + i
           |         n ← n + 1
           | [i  total]
```

$results = \begin{bmatrix} 3 \cdot 10^4 & 1.5 \cdot 10^8 \end{bmatrix}$ $largest := results_{0,0}$ $largest = 3 \cdot 10^4$

$sum := results_{0,1}$ $sum = 1.5 \cdot 10^8$

alternative solution (for this problem)

$i := 3, 6 .. 3 \cdot N$ $sum := \sum_i i$ $sum = 1.5 \cdot 10^8$

other (even better) alternative solutions (for this problem):

$3 \cdot \sum_{i=1}^{N} i = 1.5 \cdot 10^8$ or $3 \cdot \frac{N \cdot (N+1)}{2} = 1.5 \cdot 10^8$

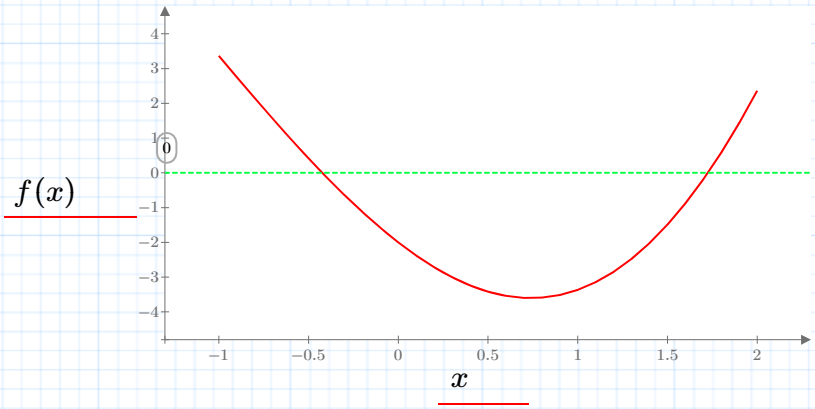
Finding roots

$f(x) := 2 \cdot x^2 - 4 \cdot \sin(x) - 2$

$x := 1$ $\text{root}(f(x), x) = 1.725$

$x := -1$ $\text{root}(f(x), x) = -0.423$

$x := -1, -0.9 .. 2$



marker added at 0 on vertical axis

Solving a set of nonlinear equations

Guess Values

$x := 1$

$y := 1$

Constraints

$x = 2 - y^2$

$y = \frac{sin(x)}{x} + x \cdot y$

Solver

$solution := \text{Find}(x, y) = \begin{bmatrix} 0.252 \\ 1.322 \end{bmatrix}$

$x := solution_0 = 0.252$ $y := solution_1 = 1.322$

Checking results (solving symbolically and plotting)

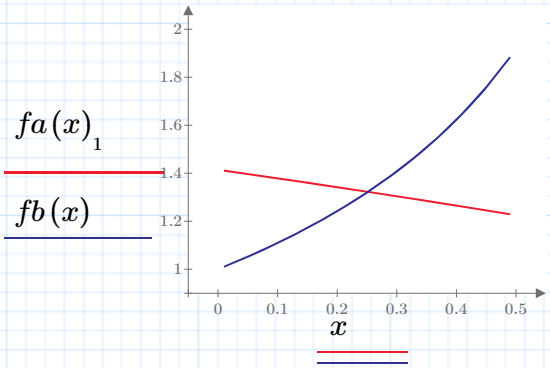
$\text{clear}_{\text{sym}}(x, y)$

$fa(x) := x = 2 - y^2 \xrightarrow{solve, y} \begin{bmatrix} \sqrt{x-2} \cdot 1\text{i} \\ -\left(\sqrt{x-2} \cdot 1\text{i}\right) \end{bmatrix}$

$fb(x) := \left(y = \frac{\sin(x)}{x} + x \cdot y\right) \xrightarrow{solve, y} -\frac{\sin(x)}{x \cdot (x-1)}$

$fa(x)_1 = 1.322$ $fb(x) = 1.322$

$x := 0.01, 0.05 .. 0.5$



Iterative calculations with subscripts

$i := 1 .. 20$ $x_0 := 1$ $y_0 := 1$

$x_i := x_{i-1} + 2$

$y_i := \frac{x_{i-1} + x_i}{2}$

$x = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \\ 11 \\ 13 \\ \vdots \end{bmatrix}$ $y = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ \vdots \end{bmatrix}$

Finding an optimal solution given constraints

$$z(x,y) := (x-1)^2 - x \cdot \sin(y)$$

Guess Values

$$x := 1$$

$$y := 1$$

Constraints

$$x > -2$$

$$x < 2 \cdot y^2 + 3$$

$$-3 < y < 5$$

Solver

$$z\_max\_loc := \text{Maximize}(z, x, y) = \begin{bmatrix} -2 \\ 1.571 \end{bmatrix}$$

$$x := z\_max\_loc_0 = -2$$

$$y := z\_max\_loc_1 = 1.571$$

$$z(x,y) = 11$$