Chapter 9 Summary
Sensors

position sensors:

- proximity sensors and switches
- potentiometer
- linear variable differential transformer (LVDT)
- digital encoder (absolute, relative)

strain gages:

gage resistance:

$$R = \frac{\rho L}{A}$$

gage resistance change:

$$\frac{dR/R}{\varepsilon_{\text{axial}}} = 1 + 2\nu + \frac{d\rho/\rho}{\varepsilon_{\text{axial}}}$$

gage factor:

$$F = \frac{\Delta R/R}{\varepsilon_{\text{axial}}}$$

gage axial strain:

$$\varepsilon_{\text{axial}} = \frac{\Delta R/R}{F}$$

Wheatstone bridge voltage:

$$V_o = V_{\text{ex}}\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right)$$

Wheatstone bridge voltage change:

$$\frac{\Delta V_o}{V_{\text{ex}}} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4} - \frac{R_2}{R_2 + R_3}$$

Wheatstone bridge resistance change:

$$\frac{\Delta R_1}{R_1} = \frac{\frac{R_4}{R_1}\left(\frac{\Delta V_o}{V_{\text{ex}}} + \frac{R_2}{R_2 + R_3}\right)}{\left(1 - \frac{\Delta V_o}{V_{\text{ex}}} - \frac{R_2}{R_2 + R_3}\right)} - 1$$

biaxial stress:

$$\sigma_x = \frac{E}{1 - \nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1 - \nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

thin-walled pressure vessel:

hoop and longitudinal stresses:

$$\sigma_x = \frac{pr}{t} \qquad \sigma_y = \frac{pr}{2t}$$

internal pressure:

$$p = \frac{t\sigma_x}{r} = \frac{tE}{r(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)$$

$$p = \frac{2t\sigma_y}{r} = \frac{2tE}{r(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)$$

<u>rectangular rosette:</u>

$$\sigma_{\max,\min} = \frac{E}{2}\left[\frac{\varepsilon_a + \varepsilon_c}{1-\nu} \pm \frac{1}{1+\nu}\sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2}\right]$$

$$\tau_{\max} = \frac{E}{2(1+\nu)}\sqrt{2(\varepsilon_a - \varepsilon_b)^2 + 2(\varepsilon_b - \varepsilon_c)^2}$$

$$\tan 2\theta_p = \frac{2\varepsilon_b - \varepsilon_a - \varepsilon_c}{\varepsilon_a - \varepsilon_c}$$

<u>temperature measurement:</u>

resistance temperature device (RTD):

$$R = R_0[1 + \alpha(T - T_0)]$$

thermistor:

$$R = R_0 e^{\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]}$$

<u>thermocouple:</u>

Seebeck effect:

$$V = \alpha(T_1 - T_2)$$

Law of Leadwire Temperatures (T_3):

$$V = V(T_1, T_2) \neq V(T_3)$$

Law of Intermediate Leadwire Metals (C):

$$V = V(A, B) \neq V(C)$$

Law of Intermediate Junction Metals (C):

$$V = V(A, B) \neq V(C)$$

Law of Intermediate Temperatures (T_1, T_2, T_3):

$$V_{13} = V_{12} + V_{23}$$

Law of Intermediate Metals (A, C, B):

$$V_{AB} = V_{AC} + V_{BC}$$

thermocouple polynomial fit:

$$T = \sum_{i=0}^{9} c_i V^i$$

$$= c_0 + c_1 V + c_2 V^2 + c_3 V^3 + c_4 V^4 + c_5 V^5 + c_6 V^6 + c_7 V^7 + c_8 V^8 + c_9 V^9$$

accelerometer:

DE: $$\frac{1}{\omega_n^2}\ddot{x}_r + \frac{2\zeta}{\omega_n}\dot{x}_r + x_r = -\frac{1}{\omega_n^2}\ddot{x}_i$$

natural frequency: $$\omega_n = \sqrt{\frac{k}{m}}$$

damping ratio: $$\zeta = \frac{b}{2\sqrt{km}}$$

input displacement: $$x_i(t) = X_i \sin(\omega t)$$

relative output displacement: $$x_r(t) = X_r \sin(\omega t + \phi)$$

amplitude ratio: $$\frac{X_r}{X_i} = \frac{(\omega/\omega_n)^2}{\left(\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right)^{1/2}}$$

phase angle: $$\phi = -\tan^{-1}\left(\frac{2\zeta(\omega/\omega_n)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$$

input acceleration: $$\ddot{x}_i(t) = -X_i\omega^2\sin(\omega t)$$

I/O ratio: $$H_a(\omega) = \frac{X_r\omega_n^2}{X_i\omega^2} = \frac{1}{\left(\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right)^{1/2}}$$

output displacement amplitude: $$X_r = \left(\frac{1}{\omega_n^2}\right)H_a(\omega)(X_i\omega^2)$$

input acceleration amplitude: $$(X_i\omega^2) = \frac{X_r\omega_n^2}{H_a(\omega)} \approx (\omega_n^2)X_r$$

total acceleration: $$\ddot{x}_i(t) = \omega_n^2 x_r(t)$$